

**Ensemble analysis and forecast
using a newly developed MCMC sampler
application to ocean colour
satellite observations**

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Context: SEAMLESS EU-H2020 project

Towards a **simplified** analysis and forecasting system based on a prior ensemble model simulation

Approach:

- Perform a **prior ensemble simulation**, with a state-of-the-art coupled circulation/ecosystem model.
- **Condition** this 4D ensemble **on ocean colour observations** to obtain the ensemble analysis and forecast.

Features:

- **Decouple** the complex models **simulations and the inversion** problem → more flexibility in the system.
- Focus the 4D inversion on a **specific region and time window**.
- No need for full controllability of the complex model (with so many state variables, when so few are observed).
No model restart from the analysis.
- The complex model is not used anymore as a direct constraint in the inverse problem, but only indirectly through the prior ensemble

Inverse problem

We **focus** on the small **4D subregion** ($10^\circ \times 7^\circ$) at $1/4^\circ$ resolution:

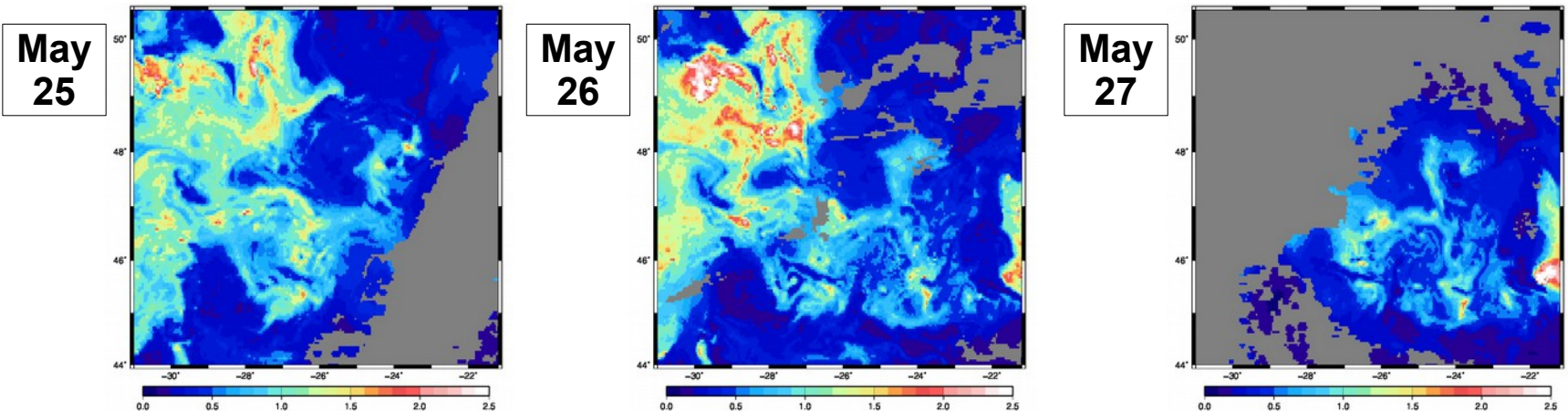
40 X 40	grid points (31°W - 21°W , 44°N - 51°N)
X 5	levels (depth: 0.5 m, 8 m, 23 m, 54 m, 108 m)
X 60	days (April 21 to June 19, 2019)
X 10	tracers (among 24 in PISCES) = $\sim 5 \times 10^6$ variables
X 40	members = $\sim 2 \times 10^8$ values

Observation system:

L3 chlorophyll product, between April 21 and June 19, 2019

Obs. error std: 30%

= $\sim 10^5$ observations



The prior ensemble simulation

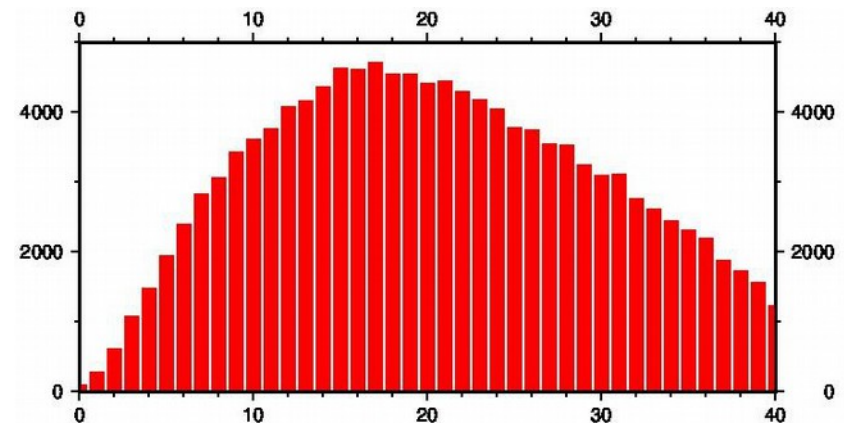
Performed in the context of the SEAMLESS EU-H2020 project.

Using a global configuration of NEMO/PISCES at $1/4^\circ$ resolution.

- 40 ensemble members.
- Outputs every 5 days for the full model state.
- Daily outputs for specific regions.

Probabilistic scores have been applied to evaluate this ensemble simulation using L3 ocean colour observations.

- Example of rank histogram for the subregion used as an example below.
- In the North Atlantic Drift: 31°W - 21°W , 44°N - 51°N , April 21 to June 19, 2019.



Method : an MCMC sampler based on the Metropolis/Hastings algorithm

Sample the posterior pdf
for the evolution of the system
 \mathbf{x} ($n \sim 5 \times 10^6$),
given observations \mathbf{y}^o ($p \sim 10^5$)

$$p(\mathbf{x}|\mathbf{y}^o) \sim p^b(\mathbf{x}) p(\mathbf{y}^o|\mathbf{x})$$

Prior ensemble
($m=40$ members)

Observation
constraint

Anamorphosis transformation
 $\mathbf{x}' = A(\mathbf{x}), \quad \mathbf{x} = A^{-1}(\mathbf{x}')$
to obtain marginally Gaussian \mathbf{x}'
(with mean=0 and variance=1):

$$p(\mathbf{x}'|\mathbf{y}^o) \sim p^b(\mathbf{x}') p[\mathbf{y}^o | A^{-1}(\mathbf{x}')]]$$

We use local
correlations only

Kept fully
general

Iterative method in 2 steps:

1. **Propose** pseudo-random perturbation of \mathbf{x}' (with cost linear in n)
→ by modulation of an ensemble member with large-scale signals
($\sim 10^{11}$ pseudo-random directions of perturbations)
→ equivalent to a localization of the prior ensemble covariance
2. **Accept/reject** according to cost function: $J^o = -\log p[\mathbf{y}^o | A^{-1}(\mathbf{x}')]]$

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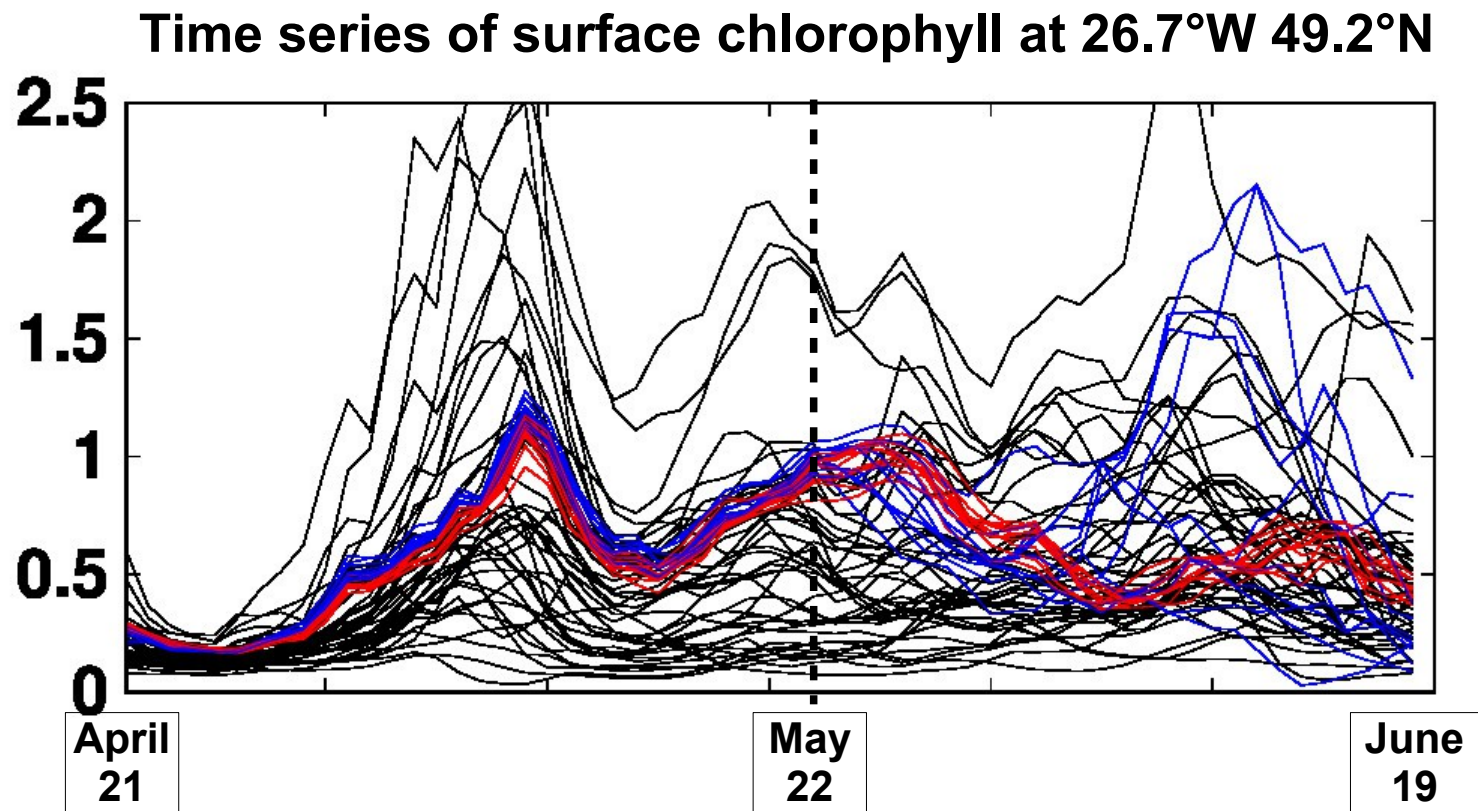
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Results from the MCMC sampler: ensemble analysis and forecast



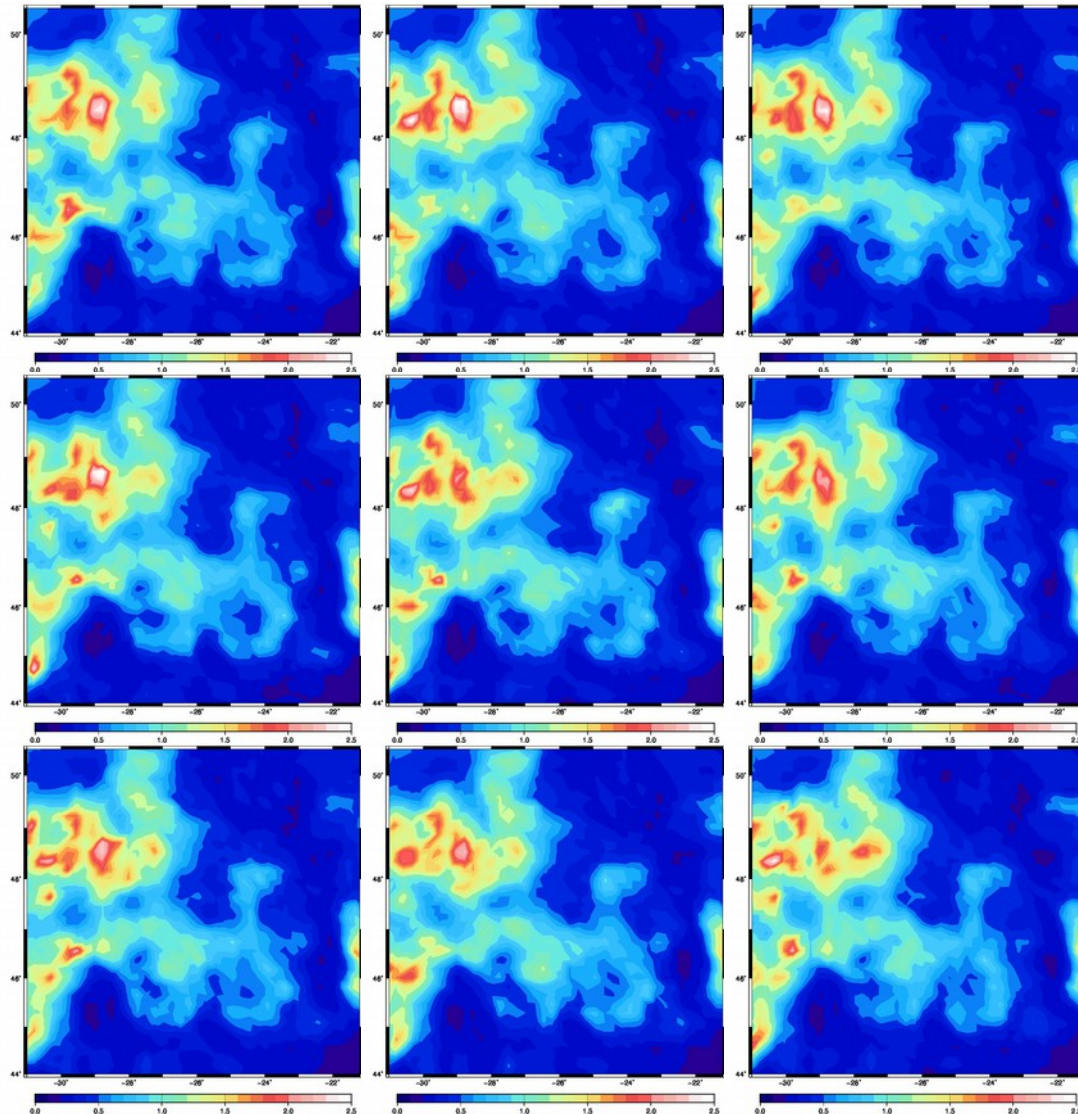
In black: prior ensemble simulations from NEMO/PISCES

In red: ensemble analysis using all L3 observations

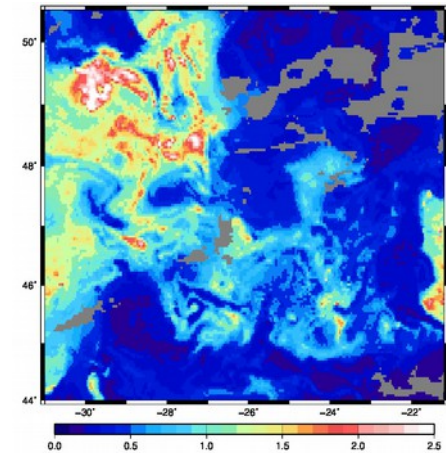
**In blue: ensemble analysis and forecast using L3 observations
until May 22 → some forecast skill for about 10 days**

[Localization scales: $\sim 0.8^\circ$ on the horizontal and ~ 10 days in time]

Results from the MCMC sampler: ensemble analysis for May 26, 2019 (using past and future observations)



L3 chlorophyll product

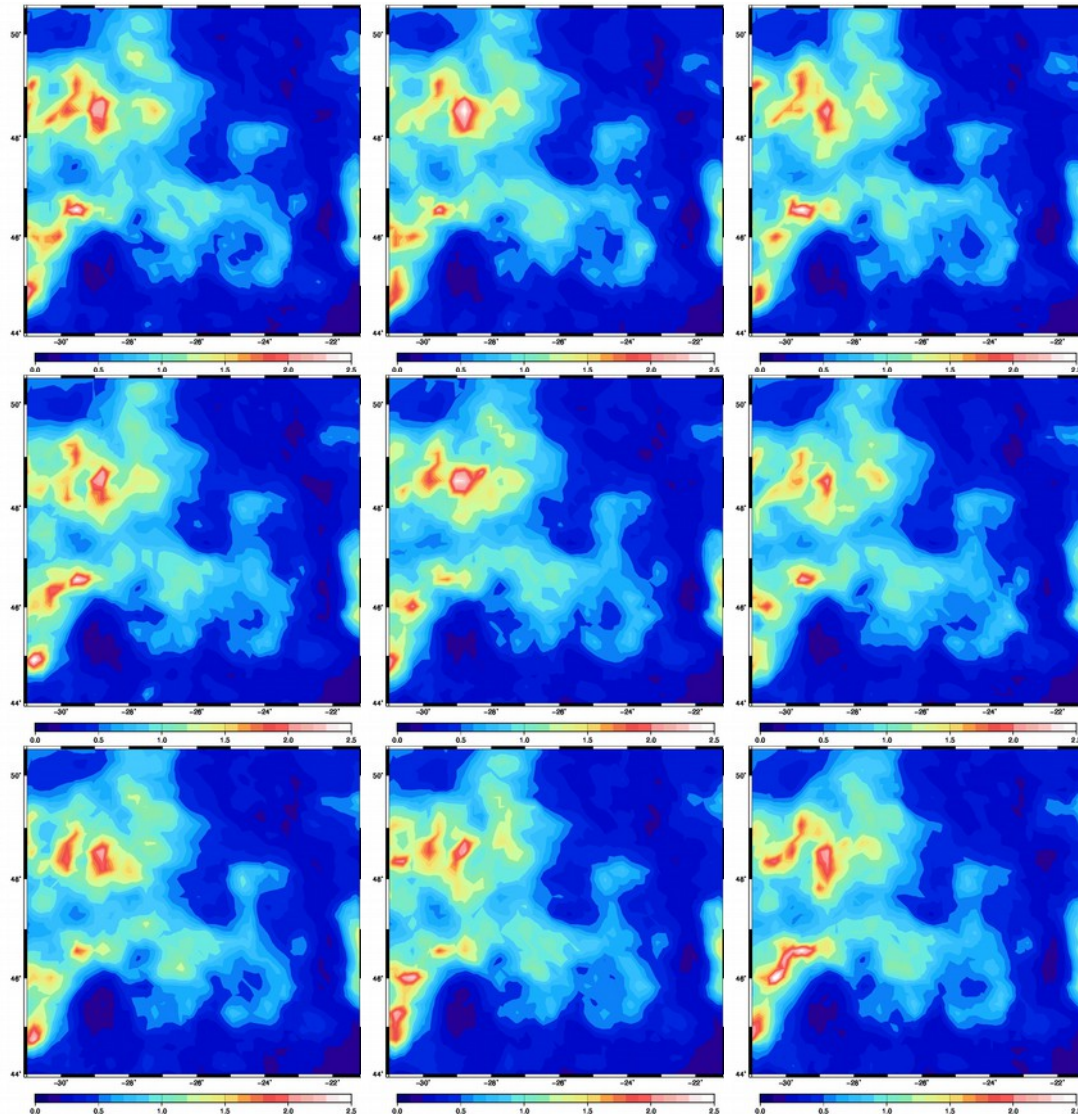


Good fit to observations
(within obs. error bar)

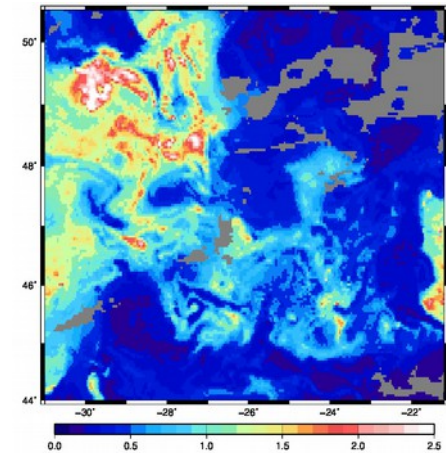
CRPS resolution:
0.121 mg/m³

Optimality score: 1.02

Results from the MCMC sampler:
ensemble analysis for May 26, 2019
(leaving out observation of May 26, 2019)



L3 chlorophyll product



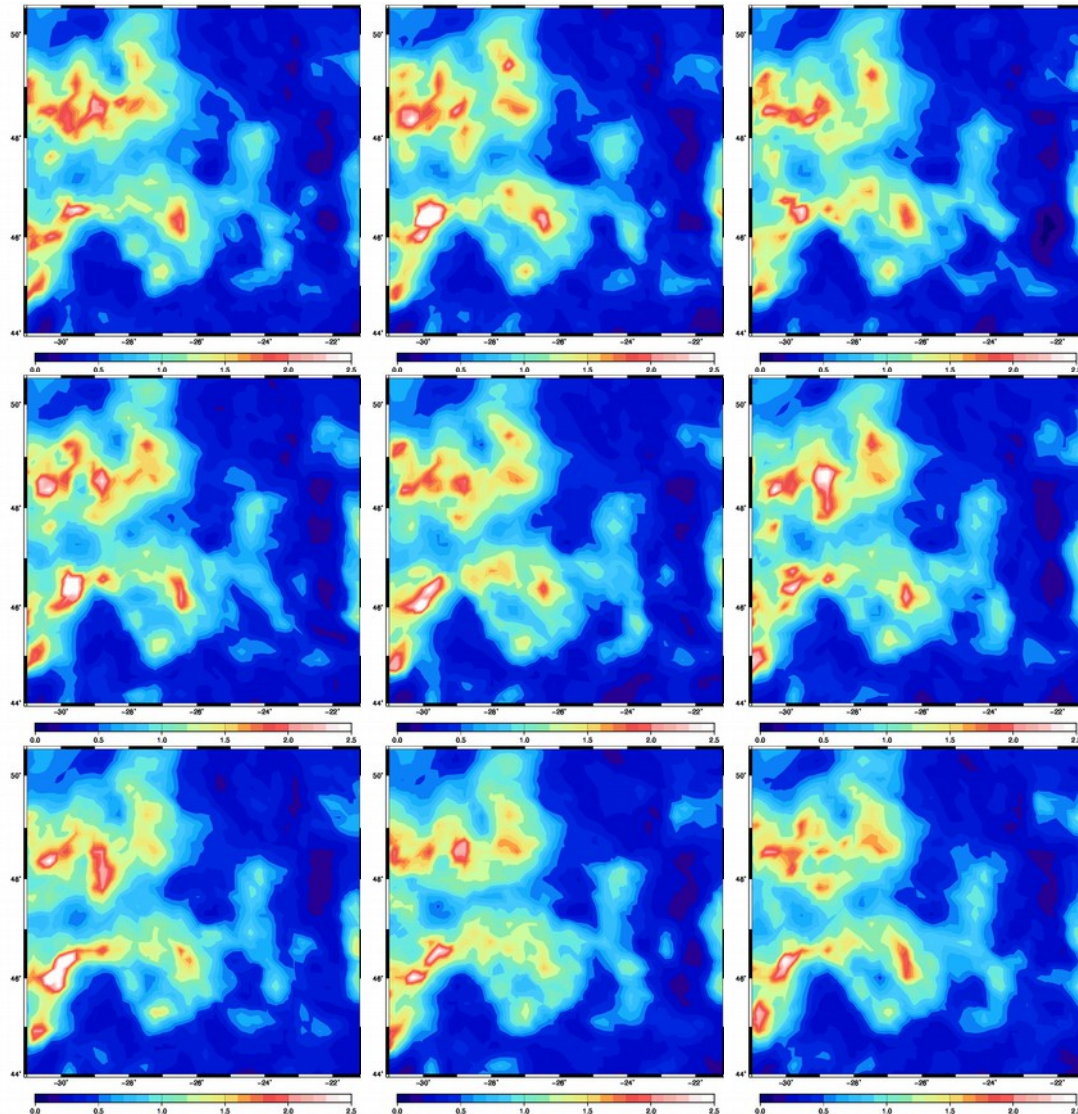
Obs. is independent.

CRPS reliability:
0.0047 mg/m³

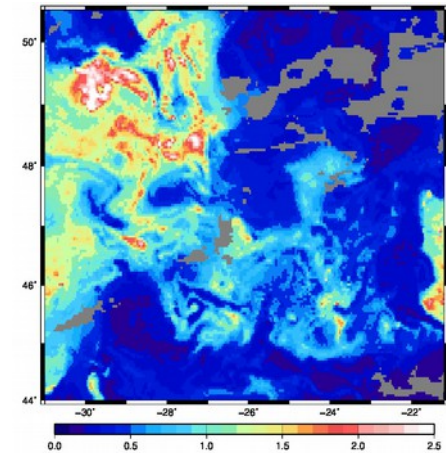
CRPS resolution:
0.132 mg/m³

Optimality score: 1.01

Results from the MCMC sampler: 1-day ensemble forecast for May 26, 2019 (i.e. using past observations only)



L3 chlorophyll product



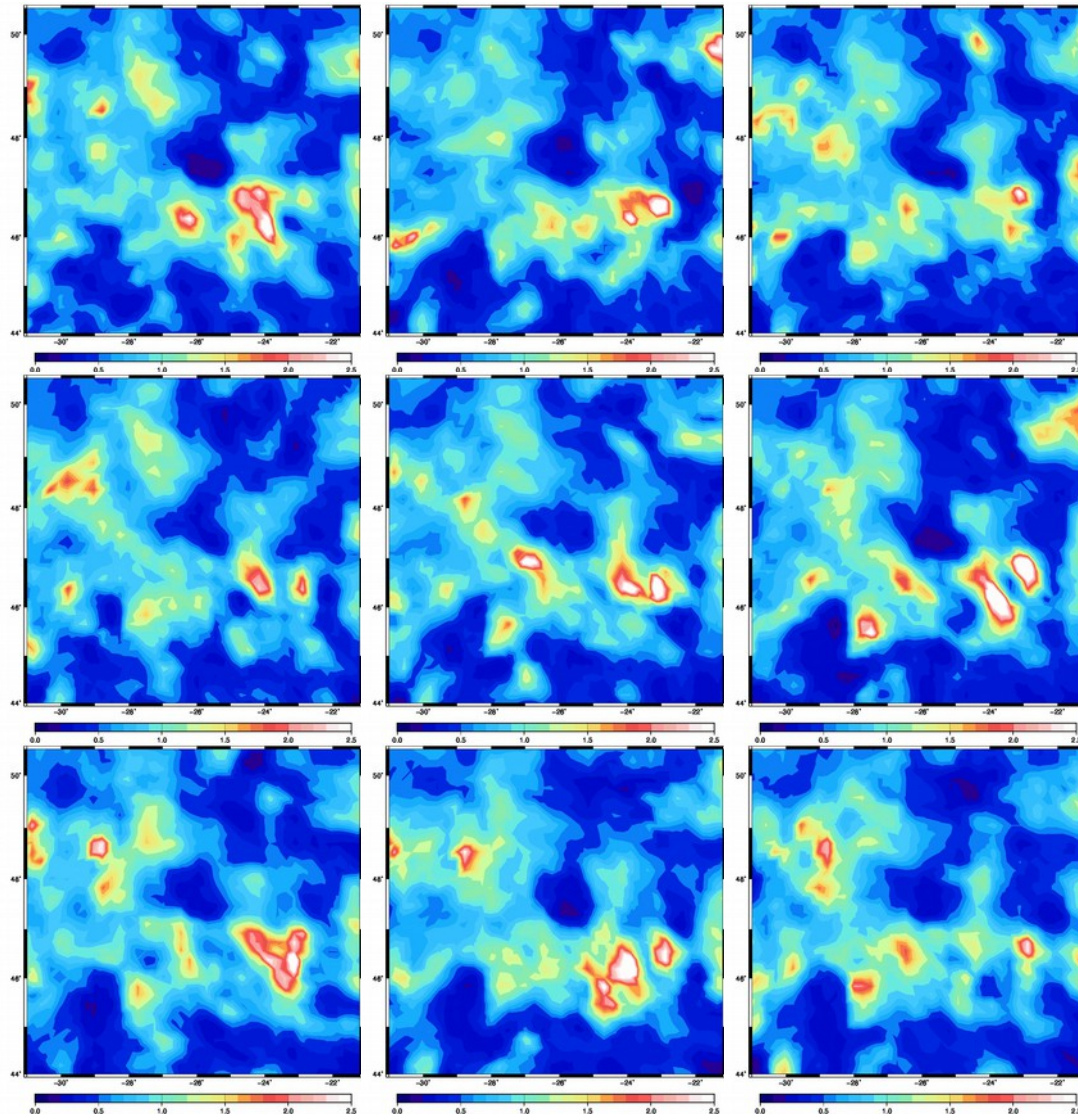
More uncertain...

CRPS reliability:
0.0089 mg/m³

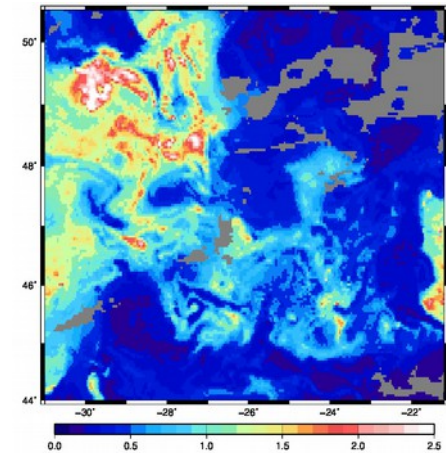
CRPS resolution:
0.145 mg/m³

Optimality score: 0.99

Results from the MCMC sampler: 4-day ensemble forecast for May 26, 2019 (i.e. using observations until May 22, 2019)



L3 chlorophyll product



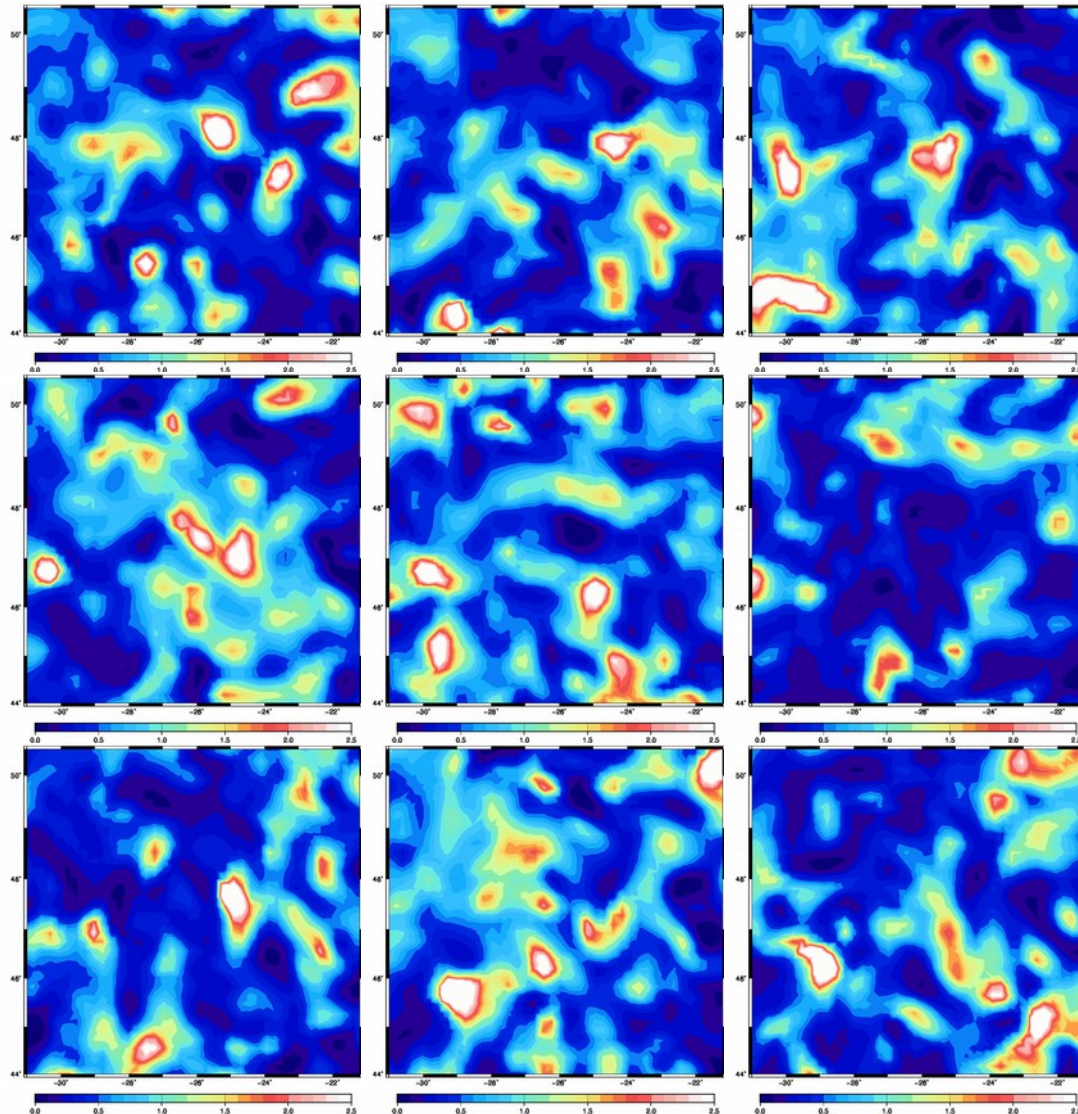
Even more uncertain...

CRPS reliability:
0.020 mg/m³

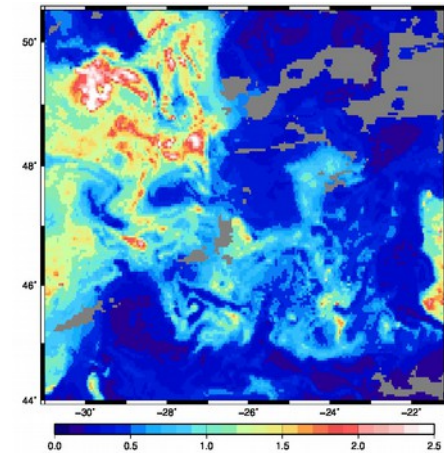
CRPS resolution:
0.191 mg/m³

Optimality score: 0.98

Results from the MCMC sampler:
prior ensemble for May 26, 2019
(i.e. from the model only, without observations)



L3 chlorophyll product



Very uncertain,
but quite reliable

CRPS reliability:
0.0077 mg/m³

CRPS resolution:
0.261 mg/m³

Conclusions

A practical method to perform **4D ensemble analyses and forecasts**

- based on prior ensemble statistics
(marginal pdfs and local rank correlations)
- coping with fully general observation constraint $p(\mathbf{y}^o|\mathbf{x})$
(nonlinear, non-Gaussian, nonlocal)

The focus is on **sampling possibilities** consistent with the observations.

To explore in SEAMLESS:

- Results for **non-observed variables** (surface and subsurface)
- Results in different regions/seasons
- How to produce a yearly solution

Perspectives (more or less remote)

Reconstruction of surface circulation:

→ from altimetric observations

→ possibly introducing a weak dynamical constraint in the cost function:

$$J^c = \frac{1}{\sigma_\psi^2} \int_{\Omega} \left[\frac{D(\Delta\psi + f)}{Dt} \right]^2 d\Omega$$

Conservation of
potential vorticity

Joint reconstruction of circulation and tracers:

→ from joint observation system, and

→ possibly introducing a joint dynamical constraint:

$$J^c = \frac{1}{\sigma_\psi^2} \int_{\Omega} \left[\frac{D(\Delta\psi + f)}{Dt} \right]^2 d\Omega + \frac{1}{\sigma_c^2} \int_{\Omega} \left[\frac{DC}{Dt} \right]^2 d\Omega$$

Including a tracer with an unknown parameter in the dynamics

$$J^c = \frac{1}{\sigma_\psi^2} \int_{\Omega} \left[\frac{D(\Delta\psi + f)}{Dt} \right]^2 d\Omega + \frac{1}{\sigma_c^2} \int_{\Omega} \left[\frac{DC}{Dt} - \mu C \right]^2 d\Omega + \frac{1}{\sigma_\mu^2} \int_{\Omega} \left[\frac{D\mu}{Dt} \right]^2 d\Omega$$

→ most simple joint physical/biogeochemical data assimilation
with joint state and parameter estimation